Maths Olympiad Contest Problems

Volume 2

Australian Edition

Exploring Maths Through Problem Solving

Contains APSMO Maths Olympiad Papers From 1996 to 2005

Contents

Preface to Australian Edition

Australasian Problem Solving Mathematical Olympiads (APSMO) Inc is proud to be affiliated with Mathematical Olympiads for Elementary and Middle Schools (MOEMS).

APSMO has been providing Mathematical Olympiads to schools throughout Australia and New Zealand since 1987. Our annual interschool Olympiads are held five times a year between May and September. There are two Divisions in the Olympiads, Division J for students up to 12 years of age and in school Year 6 or below, and Division S for students up to 14 years of age and in school Year 8 or below.

This book is the second volume to Maths Olympiad Contest Problems for Primary and Middle Schools (Australian Edition), containing the past Olympiad questions from APSMO Olympiads held between 1996 and 2005. It is an excellent resource, good for review and practice of problem solving and working mathematically techniques.

We take this opportunity to thank MOEMS for permission to reprint this text with the following modifications:

- Australian spelling
- Changes in nomenclature such as imperial to decimal measurements, American coinage to Australian coinage
- All Olympiad questions remain true to the original. In certain situations the answers may differ to the original answers, however all care has been taken to ensure that the purpose and solution methods remain unchanged. Consequently, we have continued to use 1c and 2c coins although they are no longer in use in Australia.
- Where it was not possible to change a question without altering the solution methods or intention of the question, a note has been included within the question text as clarification for students. For example: [Note: There are 3 feet in 1 yard].

Thank you to Dr Anne Prescott, lecturer in primary and secondary mathematics education at the University of Technology, Sydney, for her valuable assistance in reviewing the alterations and ensuring that the modified questions contained within this text are correct and suitable for Australian students.

Thank you also to Dr Graham Barnsley who reviewed the modifications and offered valuable advice on the presentation and interpretation of many of the questions.

Jonathan Phegan
March, 2008

Contest Problem Types

Many but not all contest problems can be categorised. This is useful if you choose to work with several related problems even if they involve different concepts.

> **KEY:** the lists below organise the problems by type and are coded by page number and problem placement on that page. For example, "Long division: J (36BD, 43C); S (84A)" refers to three problems in Division J and one in Division S, all involving long division. The three Division J problems are questions B and D on page 36 and problem C on page 43, and the Division S problem is question A on page 84.

A

Addition patterns — see *Number patterns*

Age problems: J (46B); S (109E)

Algebraic thinking: J (24D, 25E, 26C, 27E, 28C, 30B, 32D, 34E, 35D, 37C, 48AB, 53E, 54C, 57AE, 59AC, 61A, 62A, 64CD, 65C, 70B, 71B, 73B); S (79B, 81CE, 84A, 85D, 86B, 88B, 89C, 90D, 91B, 92C, 95C, 98D, 99DE, 103A, 104AD, 105AD, 106A, 108B)

Area: J (27B, 30D, 33D, 34E, 41D, 43C, 48D, 50D, 55E, 58D, 59D, 61D, 62D, 63D, 66E, 71D, 72D); S (76E, 77E, 80E, 81B, 82C, 85B, 87E, 90C, 92E, 94C, 95D, 98E, 102E, 103E, 104E)

—— **and perimeter:** J (31D, 39B, 44D, 45D, 46D, 49E, 52B, 57D, 60D, 64D); S (100D, 101E, 107D)

Arithmetic operations and properties: J (24AD, 25D, 29A, 30AC, 31A, 33B, 34A, 35A, 42A, 49A, 54A, 56C, 59A, 61D, 66A, 69A); S (78E, 81C, 86D, 86A, 89A, 91A, 92D, 93A, 97C, 99B, 102A, 104B, 106A, 107A, 109AD, 110D)

Arithmetic sequences and series — see *Number patterns*

Averages (arithmetic means): J (29E, 36A, 46E, 52C); S (76B, 92A, 96D, 97E, 102B, 103B, 107B, 109C)

—— **Weighted:** J (67E); S (81D)

B

Binary numbers: J (37D)

Business problems: J (39C, 47B, 65B); S (82D, 89D, 91C)

C

Calendar problems: J (65D, 68A); S (84C, 86A, 100E, 104C)

Certainty problems: J (57C); S (87A)

Circles: J (24C, 56D); S (79E, 80AE, 84E, 90C, 94C, 98E, 106D)

Clock problems: J (26A, 28D, 31C, 42C, 46C, 52A, 71E); S (83D, 91E, 110E)

Coin problems: J (28C, 45B, 51E); S (98B)

Combinations: J (24C, 30E, 56B, 63C, 64B); S (80A, 84E, 106B)

Congruent figures: J (26B, 29B, 30D, 39D, 40D, 44D, 45D, 47C, 49E, 51B, 53D, 62D, 65E, 68D, 69BD); S (76E, 77A, 81B, 82C, 90C, 92E, 93D, 94C, 97E, 98E)

Introduction

For the Reader

This book was written for both the participants in the Australasian Problem Solving Mathematical Olympiads and their advisors. It is suitable for mathletes who wish to prepare well for the contests, students who wish to develop higher-order thinking, and teachers who wish to develop more capable students. All problems were designed to help students develop the ability to think mathematically, rather than to teach more advanced or unusual topics. While a few problems can be solved using algebra, nearly all problems can be solved by other, more elementary, methods. In other words, the fun is in devising non-technical ways to solve each problem.

The 425 Maths Olympiad contest problems contained in this book are organised into 17 sets of five contests each, every set representing one year's competition. Ten of the sets were created for Division J for students in Years 4-6, and the other seven for Division S for students in Years 7-8. These problems exhibit varying degrees of difficulty and were written for contests between 1996 and 2005, inclusive.

The introduction is arranged into three parts. Sections 1 to 5, written for all readers, contain discussions of problem solving in general. Sections 6 to 8 offer many suggestions for getting the most out of this book. Sections 9 to 14, designed for the advisor, called the **P**erson-**I**n-**C**harge-of-the-**O**lympiads (PICO), include recommendations related to the various aspects of organising a Maths Olympiad program.

1. How to Use This Book

Establishing a Study Schedule

A little learning every day is more effective than large chunks of learning once a month for two reasons. The mind needs time to absorb each new thought, and constant practice allows frequent review of previously learned concepts and skills. Together, these foster retention. Try to spend 10 or 15 minutes daily doing one or two problems. This approach should help you minimise the time needed to develop the ability to think mathematically.

You might want to track growth over time by recording success rates for these problem sets. Since you are probably changing the way you think about mathematics, your growth needs time to become apparent. Before long, you are likely to find solving problems intensely and increasingly rewarding.

Set 1: Olympiad 1

1A
3 Minutes

What is the value of the following, in simplest terms?

$$(20 \times 24 \times 28 \times 32) \div (10 \times 12 \times 14 \times 16)$$

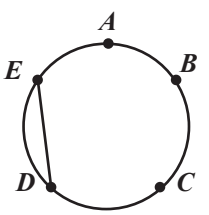
1B
5 Minutes

Roni starts with the number 5 and counts by 8s. This results in the sequence 5, 13, 21, 29, 37, and so on.

What is the twenty-fifth number in the sequence?

1C
6 Minutes

A line segment (such as *ED* as shown) that connects any two points of a circle is called a *chord of the circle*. How many different chords, including *ED,* can be drawn using only points *A*, *B*, *C*, *D*, and *E*?
(*Note: ED is the same as DE)*



1D
5 Minutes

A represents a counting number. Find the value of *A* if:

$$\frac{A + A}{A \times A} = \frac{1}{3}$$

1E
6 Minutes

Ben and Jerry start with the same number of trading cards. After Ben gives 12 of his cards to Jerry, Jerry then has twice as many cards as Ben does.

How many cards did Ben have at the start?

Hints: Division J

Set 1, Olympiad 1

1A. Compare each number in the first set of parentheses with its counterpart in the second set.
1B. What would happen if each number were increased by 3?
1C. How could you use a tree diagram to answer this question?
1D. Why is A a multiple of 3?
1E. Draw a stack to represent Ben's starting collection of $x + 12$ cards.

Set 1, Olympiad 2

2A. Suppose Marty gave the 10 pogs to a third person, who then gave them to Jen.
2B. Draw a picture of the box made from 2-cm cubes.
2C. What are the possible values for B? How does the number of places in the partial products affect the choices?
2D. A remainder of 4 means that each answer must be a divisor of what number?
2E. Suppose only adults buy tickets.

Set 1, Olympiad 3

3A. What time is it 24 hours from now? 48 hours?
3B. How many cubes are in each stack, or in each layer?
3C. How many games did the Panthers lose?
3D. Suppose there are three more students. How many students would be left over then?
3E. Draw a diagram, letting a box represent one-third of the number.

Set 1, Olympiad 4

4A. What does $\frac{4}{5}$ of 100 mean?
4B. Find the area of one square. How many different ways can you find to solve this problem?
4C. Make an organised list of factors, pairing each with its cofactor.
4D. How far will the car travel in 3 minutes?
4E. How much should a purchase of 5 pens and 5 pencils cost?

Set 1, Olympiad 5

5A. A terminal zero is produced by multiplying any even number by 5.
5B. Don't forget the squares that are tilted.
5C. What is the largest number of 25c coins possible?
5D. Draw the clock.
5E. Work backwards.